MODIFIED SIGNOMIAL GEOMETRIC PROGRAMMING (MSGP) AND ITS APPLICATIONS

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ABSTRACT

A "signomial" is a mathematical function, contains one or more independent variables. Richard J. Duffin and Elmor L. Peterson (1967) introduced the term "signomial". Signomial geometric programming (SGP) optimization technique often provides a much better mathematical result of real-world nonlinear optimization problems. In this research paper, we have proposed unconstrained and constrained signomial geometric programming (SGP) problem with positive or negative integral degree of difficulty. Here a modified form of signomial geometric programming (MSGP) has been developed and some theorems have been derived. Finally, these are illustrated by proper examples and applications.

Keywords: Polynomial; Constrained problem; Signomial function; Modified Signomial Geometric Programming

1. INTRODUCTION

Geometric programming (GP) method is a relatively new optimization method to solve a various types of non-linear programming problem. The idea of geometric programming technique was fast developed by Duffin, Peterson and Zener. Since 1960's geometric programming (GP) has been known and used in various field like as operations management, industrial engineering, structural problem etc..

In the late 1960s and early 1970s the term "signomial" was introduced by Duffin and Peterson (1967) in their seminal joint work on general algebraic optimization. A recent introductory exposition is non-linear optimization problems. Although non-linear optimization problems with constraints and/or objectives defined by signomials are normally harder to solve than those defined by only posynimials (because unlike posynomials, signomials are not guaranteed to be globally convex).

A signomial optimization geometric programming technique often provides a much more accurate and better mathematical representation of real-world non-linear optimization problems.

Passy and Wilde (1967) and Blau and Wilde (1969) generalized some of the prototype concepts and theorems in order to treat signomial geometric programming problems. In 1988 general type of signomial geometric programming has been done by Charnes and Cooper, who proposed methods for approximating signomial geometric programs with prototype geometric programs. Recently Islam and Roy (2005) has been demonstrated modified geometric programming problem and its application.

A "signomial" is a mathematical (algebraic) function of one or more independent function. "signomial" is one of the most easily thought of as an algebraic extension of multi-dimensional polynomials (posynomial) an extension that permits exponents to be arbitrary real numbers (rather than just non negative integers) while requiring the independent variables to be strictly positive .

In this research paper we propose unconstrained / constrained signomial geometric programming (SGP) and modified form of signomial geometric programming (SGP) problem with positive or negative integral of difficulty. Modified form of signomial geometric programming has been demonstrated and some

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theorems have been presented here. Finally, these are illustrated by proper real type of numerical examples and applications.

2. SOME BASIC CONCEPTS

Monomial: The word 'monomial' comes from the Latin word, *mono* meaning only one and *mial* meaning term. So a monomial is an "expression in algebra that contains only one term". The term monomial as used in the context of geometric programming (GP) technique is similar to, but differs from the standard definition of monomial used in algebra. In mathematics a monomial can be a constant, a variable or the product of one or more constant and variables but the exponent of variables cannot have a negative or fractional. Throughout the exponent can be any real number, including fractional and negative.

So if $x_1, x_2, \ldots\ldots\ldots\ldots, x_n$ denote n real positive variable, then a real valued function f of x, in the form

$$F(x) = cx_1{}^{a_1}x_2{}^{a_2}\ldots\ldots\ldots x_n{}^{a_n},$$

where c > 0 and $a_i \in \mathbf{R}$ is called monomial algebraic function.

Polynomial: The word 'polynomial' comes from the Latin word, *polyno* meaning many and *mial* meaning term. So a polynomial is an "expression in algebra that contains many terms i.e., many monomial".

So sum of one or more monomial function, i.e., any function of the form

$$F(x) = \sum_{i=1}^{i} c_i\, x_1{}^{a_{1i}}x_2{}^{a_{2i}}\ldots\ldots\ldots x_n{}^{a_{ni}},$$

is called polynomial function or in simply called polynomial.

Posynomial: If coefficients $c_i > 0$ $(i = 1,2,\ldots\ldots n)$, then a polynomial function is called a posynomial function.

So sum of one or more monomials, i.e., any function of the form

$$F(x) = \sum_{i=1}^{i} c_i\, x_1{}^{a_{1i}}x_2{}^{a_{2i}}\ldots\ldots\ldots x_n{}^{a_{ni}},$$

where $c_i > 0$, is called posynomial function or simply posynomial.

Note 1: The term posynomial function is mean to suggest a combination of positive and polynomial function. So Positive+Polynomial = Posynomial.

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Signomial: A signomial function is differs from a posynomial function in that the coefficient need not be positive. Let X is a vector of real and positive numbers.

$$X = (x_1, x_2, \ldots \ldots, x_m)^T$$

Then the signomial function has the form as follows

$$g_k(x) = f(x_1, x_2, \ldots \ldots, x_m) = \sum_{i=1}^{k} \sigma_i c_i \prod_{j=1}^{m} x_j^{a_{ij}}$$

Where c_i = absolute value of coefficient,

σ_i = sign of coefficient (+1 or -1).

Signomial functions are closed under addition, subtraction, multi-plication and scaling of basic mathematical operation.

Example 1: $g_k(x) = 2x_1^2 x_2^{-1} - 5x_1 x_2$

For this signomial

$\sigma_1 = +1, \sigma_2 = -1,$

$c_1 = 2, c_2 = 5.$

Table 1: Compare between monomial, polynomial and posynomial:

Monomial	Polynomial	Posynomial
1) Contains only one term. 2) Addition of two or more monomials is not monomial. 3) Subtraction of two or more monomials is not monomial. 4) Multiplication of two or more monomials is a monomial. 5) Division of a monomial by one or more other monomials is a monomial. 6) A monomial is of the form, $F(x) = cx_1^{a_1} x_2^{a_2} \ldots \ldots x_n^{a_n}$, c> 0. 7) Examples: 5, 2x, $x^2 y^4$.	1) Contains one or more term. 2) Addition of two or more polynomials is polynomial. 3) Subtraction of two or more polynomials is a polynomial. 4) Multiplication of two or more polynomials is a polynomial. 5) Division of a polynomial by one or more other monomials is a polynomial. 6) A polynomial is of the form, F(x)= $\sum_{i=1}^{i} c_i x_1^{a_{1i}} x_2^{a_{2i}} \ldots \ldots x_n^{a_{ni}}$ 7) Examples: 5, 2x+y, $x^2 - y^4$.	1) Contains one or more term. 2) Addition of two or more posynomials is posynomial. 3) Subtraction of two or more posynomials is not posynomial. 4) Multiplication of two or more posynomials is a posynomial. 5) Division of a posynomial by one or more other monomials is a posynomial. 6) A posynomial is of the form, F(x)= $\sum_{i=1}^{i} c_i x_1^{a_{1i}} x_2^{a_{2i}} \ldots \ldots x_n^{a_{ni}}$, $c_i > 0$. 7) Examples: 5, 2x+y, $x^2 + y^4$.

3.   UNCONSTRAINED PROBLEM

3.1.   Signomial geometric programming (SGP) problem:

3.1.1. Primal program:

A primal unconstrained signomial geometric programming (SGP) problem is of the following form

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Minimize $g_0(x_1, x_2, \ldots\ldots\ldots\ldots, x_m)$ (3.1)

Subject to $x_j > 0,$ j = 1, 2,……,m.

Where $g_0(x) = \sum_{i=1}^n \sigma_i c_i \prod_{j=1}^m x_j{}^{a_{ij}}$.

Here c_i is absolute value of coefficient, σ_i is sign of coefficient (+1 or -1) and a_{ij} be any real number. It is unconstrained signomial geometric programming (SGP) problem with the degree of difficulty (DD) $= n - (m + 1)$.

3.1.2. Dual program:

The dual geometric programming (DGP) problem of the given primal geometric programming (GP) problem is

Maximize $v(\delta) = \zeta_0 \left[\prod_{i=1}^n (\frac{c_i}{\delta_i})^{\sigma_i \delta_i} \right]^{\zeta_0}$ (3.2)

Subject to $\sum_{i=1}^n \sigma_i \delta_i = \zeta_0,$

$\qquad\qquad \sum_{i=1}^n \sigma_i a_{ij} \delta_i = 0, \qquad\qquad j = 1,2, \ldots\ldots\ldots, m$

$\qquad\qquad \delta_i > 0,$

Case I: If n>m+1, (i.e. DD>0) then the dual program (DP) presents a system of linear equations for the dual variables. Here the number of dual variables is more than the number of linear equations. Here more optimal solutions of dual variable vector exist. In order to find an optimal solution of the given dual program (DP), we need to use some algorithmic techniques.

Case II: If n< m+1, (i.e. DD<0) then the dual program (DP) presents a system of linear equations for the dual variables. Here the number of dual variables is less than the number of linear equations. In this special case generally no solution vector exists for the dual variables. However, using some special types of techniques that is Least Square (LS) or Min-Max (MM) method one can get an approximate solution for this type of problem.

Moreover from the primal-dual relation,

$\qquad\qquad c_i \prod_{j=1}^m x_j{}^{*a_{ij}} = \zeta_0 \delta^*{}_i \, v(\delta^*, \lambda^*).$ (3.3)

Taking logarithms in (3.3), T$_0$ log-linear simultaneous equations are transformed as

$$\sum_{j=1}^{n} \alpha_{ij}(\log x_j) = \log\left(\frac{\varsigma_0 \delta_i^* v^*(\delta^*, \lambda^*)}{c_i}\right), \quad (i=1,2,\ldots,n). \qquad (3.4)$$

It is a system of n linear equations in t_j (=log x_j) for j=1,2,...,m.

Note 2: A Weak Duality theorem say that $g_0(x) \geq v(\delta)$ for any primal-feasible x and dual-feasible δ but this inequality is not true of the pseudo-dual signomial geometric programming (SGP) optimization problem.

Corollary 1: When the values of σ_i $(i = 1,2,\ldots.n)$ are 1, then a signomial geometric programming (SGP) problem transform to ordinary geometric programming (GP) problem.

3.1.3. Theorem: When the values of σ_i $(i = 1,2,\ldots.n)$ are 1, then $g_o(x) \geq v(\delta)$ (Primal−Dual Inequality).

Proof

The expression for $g_0(x)$ can be written as

$$g_o(x) = \sum_{i=1}^{n} \delta_k \left(\frac{c_i \prod_{j=1}^{m} x_j^{\alpha_{kj}}}{\delta_k}\right).$$

Here the weights are $\delta_1, \delta_2, \ldots\ldots\ldots, \delta_n$ and positive terms are $\frac{c_1 \prod_{j=1}^{m} x_j^{\alpha_{1j}}}{\delta_1}$,

$\frac{c_2 \prod_{j=1}^{m} x_j^{\alpha_{2j}}}{\delta_2}, \ldots\ldots\ldots, \frac{c_n \prod_{j=1}^{m} x_j^{\alpha_{nj}}}{\delta_n}$.

Now applying the AM−GM inequality, we get

$$\left(\frac{c_1 \prod_{j=1}^{m} x_j^{\alpha_{1j}} + c_2 \prod_{j=1}^{m} x_j^{\alpha_{2j}} + \ldots + c_n \prod_{j=1}^{m} x_j^{\alpha_{nj}}}{(\delta_1 + \delta_2 + \cdots + \delta_n)}\right)^{(\delta_{01}+\delta_{02}+\cdots+\delta_n)}$$

$$\geq \left(\left(\frac{c_1 \prod_{j=1}^{m} x_j^{\alpha_{1j}}}{\delta_1}\right)^{\delta_1}\left(\frac{c_2 \prod_{j=1}^{m} x_j^{\alpha_{2j}}}{\delta_2}\right)^{\delta_2} \ldots \left(\frac{c_n \prod_{j=1}^{m} x_j^{\alpha_{nj}}}{\delta_n}\right)^{\delta_n}\right)$$

Or $\quad \left(\frac{g_0(x)}{\sum_{i=1}^{n} \delta_i}\right)^{\sum_{i=1}^{n} \delta_i} \geq \prod_{i=1}^{n}\left(\frac{c_i \prod_{j=1}^{m} x_j^{\alpha_{nj}}}{\delta_i}\right)^{\delta_i} \qquad [as \sum_{i=1}^{n} \delta_k = 1]$

Or $\quad g_0(x) \geq \left(\frac{c_i}{\delta_{0k}}\right)^{\sum_{i=1}^{n} \delta_i} \prod_{j=1}^{m} x_j^{\sum_{i=1}^{n} \alpha_{ij}\delta_i}$

Or $\quad g_0(x) \geq \prod_{i=1}^{n}\left(\frac{c_i}{\delta_i}\right)^{\delta_i} \prod_{j=1}^{m} x_j^{\sum_{i=1}^{n} \alpha_{ij}\delta_i}$

$\qquad\qquad = \prod_{i=1}^{n}\left(\frac{c_i}{\delta_i}\right)^{\delta_i} \qquad\qquad\qquad [as \sum_{k=1}^{T_0} \alpha_{0kj}\delta_{ok} = 0]$

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$$= v(\delta)$$

i.e., $g_0(x) \geq v(\delta)$.

This completes the proof.

Example (Unconstrained SGP problem) 2:

Min $Z(x, y) = 10x + 5xy - 10x^3 y$

Subject to   x, y > 0.

Solution:

Assume $\zeta_0 = 1$,

Corresponding dual of the primal geometric problem is,

Maximize v(δ)$= (\frac{10}{\delta_1})^{\delta_1}(\frac{5}{\delta_2})^{\delta_2}(\frac{10}{\delta_3})^{-\delta_3}$ (3.5)

Subject to $\delta_1 + \delta_2 - \delta_3 = 1$,

$\delta_1 + \delta_2 - 3\delta_3 = 0$,

$\delta_2 - \delta_3 = 0$,   (3.6)

$\delta_1, \delta_2, \delta_3 > 0$.

From (6), $\delta_1 = 1, \delta_2 = \frac{1}{2}$ and $\delta_3 = \frac{1}{2}$. Putting the value of δ_1, δ_2 and δ_3 in (3.5), the corresponding optimal dual value is $v(\delta) = 7.071$.

So for primal optimal decision variables, the following equations found

$10x = \delta_1 v(\delta) = 1 \times 7.071$,

$5xy = \delta_2 v(\delta) = \frac{1}{2} \times 7.071$,

$10x^3 y = \delta_3 v(\delta) = \frac{1}{2} \times 7.071$.

Taking logarithm of each side yields equations which are linear in the logarithms of decision the primal variables

In 10 + In x $=$ In 7.071,

In 5 + log x + log x $=$ In 3.535,   (3.7)

In 10 + 3log x + log y $=$ In 3.535.

From (3.7), optimal solutions are

$x = 0.707,$

$y = 1.$

And corresponding optimum value, Z(x, y) = 7.071.

3.2.   Modified signomial GP problem:

3.2.1. Primal program:

A primal unconstrained modified signomial geometric programming (SGP) problem is of the form

Minimize $g_0(x_{lj})$

Subject to $x_{lj} > 0$, j = 1, 2,……,m.

Where $g_0(x) = \sum_{l=1}^{n} \sum_{i=1}^{k} \sigma_{li} c_{li} \prod_{j=1}^{m} x_{lj}{}^{a_{lij}}.$ (3.8)

Here c_{li} is absolute value of coefficient, σ_{li} is sign of coefficient (+1 or − 1) and a_{lij} be any real number. It is unconstrained signomial MGP problem with the degree of difficulty (DD) $= nk − (nm + n)$.

3.2.2. Dual program:

Dual geometric program (GP) problem of the given primal GP problem is

Maximize $\zeta_0 \left[\prod_{l=1}^{n} \prod_{i=1}^{k} (\frac{c_{li}}{\delta_{li}})^{\sigma_{li}\delta_{li}} \right]^{\zeta_0}$ (3.9)

Subject to $\sum_{i=1}^{k} \sigma_{li}\delta_{li} = \zeta_0,$ $(l = 1,2, … … … . , n).$

$\sum_{i=1}^{k} \sigma_{li}a_{lij}\delta_{li} = 0,$ $(l = 1,2, … … … . , n; j = 1,2, … … … . , m).$

$\delta_{li} > 0,$

Case I: If nk≥nm+n, (i.e. DD>0) then the dual program (DP) presents a system of linear equations for the dual variables. Here the number of dual variables is greater than the number of linear equations. More solutions of dual variable vector exist. In order to find an optimal solution of dual program (DP), we need to use some algorithmic methods.

Case II: If nk<nm+n, (i.e. DD<0) then the dual program (DP) presents a system of linear equations for the dual variables. Here the number of dual variables is less than

the number of linear equations. In this case generally no solution vector exists for the dual variables. However, using the Least Square (LS) or the Min-Max (MM) technique one can get an approximate solution for this non-linear system.

Furthermore the primal-dual relation is

$$c_{li} \prod_{j=1}^{m} x_{lj}{}^{\alpha_{lij}} = \zeta_0 \delta^*{}_{li} \sqrt[n]{v(\delta^*)}., \ (l = 1,2,\dots\dots\dots,k; i = 1,2,\dots\dots\dots,n). \tag{3.10}$$

Note 3: A Weak Duality theorem would say that $g_0(x_{lj}) \geq n\sqrt[n]{v(\delta)},$ for any primal-feasible x and dual-feasible δ but this is not true of the pseudo-dual signomial geometric programming (SGP) problem.

Corollary 2: When the values of σ_{li} is 1, then a modified signomial geometric programming (MSGP) problem transform to ordinary modified geometric programming problem.

3.2.3. Theorem: When σ_i is 1, then $g_0(x_{ij}) \geq n\sqrt[n]{v(\delta)}$ (Primal- Dual Inequality).

Proof

The expression for $g_0(x_{ij})$ can be written as

$$g_0(x_{ij}) = \sum_{l=1}^{n} \sum_{i=1}^{k} \delta_{ik} \left(\frac{c_{li} \prod_{j=1}^{m} x_{ij}{}^{\alpha_{lij}}}{\delta_{ik}} \right).$$

Here the weights are $\delta_{l1}, \delta_{l2}, \dots\dots\dots, \delta_{lk}$ and positive terms are $\dfrac{c_{l1} \prod_{j=1}^{m} x_j{}^{\alpha_{l1j}}}{\delta_{l1}},$

$\dfrac{c_{l2} \prod_{j=1}^{m} x_j{}^{\alpha_{l2j}}}{\delta_{l2}}, \dots\dots\dots, \dfrac{c_{lk} \prod_{j=1}^{m} x_j{}^{\alpha_{lnj}}}{\delta_{lk}}$.

Now applying AM-GM inequality, we get

$$\left(\frac{\sum_{l=1}^{n}(c_{l1} \prod_{j=1}^{m} x_{ij}{}^{\alpha_{l1j}} + c_{l2} \prod_{j=1}^{m} x_{ij}{}^{\alpha_{l2j}} + \dots + c_{ln} \prod_{j=1}^{m} x_{ij}{}^{\alpha_{lkj}})}{\sum_{l=1}^{n}(\delta_{l1} + \delta_{l2} + \dots + \delta_{lk})} \right)^{\sum_{i=1}^{n}(\delta_{l1}+\delta_{l2}+\dots+\delta_{lk})}$$

$$\geq \sum_{l=1}^{n} \left(\left(\frac{c_{l1} \prod_{j=1}^{m} x_{ij}{}^{\alpha_{i1j}}}{\delta_{l1}} \right)^{\delta_{l1}} \left(\frac{c_{l2} \prod_{j=1}^{m} x_{ij}{}^{\alpha_{l2j}}}{\delta_{l2}} \right)^{\delta_{l2}} \dots \left(\frac{c_{lk} \prod_{j=1}^{m} x_{ij}{}^{\alpha_{lkj}}}{\delta_{lk}} \right)^{\delta_{lk}} \right)$$

Or $\quad \left(\dfrac{g_0(x_{ij})}{\sum_{l=1}^{n} \sum_{i=1}^{k} \delta_{li}} \right)^{\sum_{l=1}^{n} \sum_{i=1}^{k} \delta_{li}} \geq \prod_{l=1}^{n} \prod_{i=1}^{k} \left(\dfrac{c_{li} \prod_{j=1}^{m} x_{ij}{}^{\alpha_{lij}}}{\delta_{ik}} \right)^{\delta_{ik}}$

Or $\quad \left(\dfrac{g_0(x_{ij})}{n} \right)^n \geq \prod_{l=1}^{n} \left(\dfrac{c_{li}}{\delta_{lk}} \right)^{\sum_{i=1}^{k} \delta_{lk}} \prod_{j=1}^{m} x_{ij}{}^{\sum_{i=1}^{k} \alpha_{lij}\delta_{li}} \qquad [as \ \sum_{i=1}^{k} \delta_{li} = 1]$

$$= \prod_{l=1}^{n} \prod_{i=1}^{k} \left(\dfrac{c_{li}}{\delta_{li}} \right)^{\delta_{li}} \prod_{j=1}^{m} x_{ij}{}^{\sum_{i=1}^{k} \alpha_{lij}\delta_{li}}$$

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Or $\quad (\frac{g_0(x_{ij})}{n})^n \geq \prod_{l=1}^{n} \prod_{i=1}^{k} \left(\frac{c_{li}}{\delta_{li}}\right)^{\delta_{li}}$ $\qquad\qquad [as\ \sum_{i=1}^{k} \alpha_{lij}\delta_{li} = 0]$

$\qquad = v(\delta)$

i.e., $\quad g_0(x_{ij}) \geq n\ \sqrt[n]{v(\delta)}$.

This completes the proof.

Example (Unconstrained MSGP problem) 3:

Min Z(x)= $10x_{11} + 5x_{11}x_{12} - 10x_{11}{}^3x_{12} + 6x_{21} + 10x_{21}x_{22} - 5x_{21}{}^3x_{22}$

Subject to $\quad x_{11}, x_{12}, x_{21}, x_{22} > 0$.

Solution:

Assume $\zeta_0 = 1$,

Corresponding dual of the given primal problem is

Maximize $v(\delta) = (\frac{10}{\delta_1})^{\delta_1}(\frac{5}{\delta_2})^{\delta_2}(\frac{10}{\delta_3})^{-\delta_3}(\frac{6}{\delta_{01}})^{\delta_{01}}(\frac{10}{\delta_{02}})^{\delta_{02}}(\frac{5}{\delta_{03}})^{-\delta_{03}}$ $\qquad\qquad$ (3.11)

Subject to $\quad \delta_1 + \delta_2 - \delta_3 = 1$,

$\qquad\qquad \delta_1 + \delta_2 - 3\delta_3 = 0$,

$\qquad\qquad \delta_2 - \delta_3 = 0$, $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (3.12)

$\qquad\qquad \delta_{01} + \delta_{02} - \delta_{03} = 1$,

$\qquad\qquad \delta_{01} + \delta_{02} - 3\delta_{03} = 0$,

$\qquad\qquad \delta_{02} - \delta_{03} = 0$,

$\qquad\qquad \delta_1, \delta_2, \delta_3, \delta_{01}, \delta_{02}, \delta_{03} > 0$.

From (3.12), $\delta_1 = 1$, $\delta_2 = \frac{1}{2}$, $\delta_3 = \frac{1}{2}$, $\delta_{01} = 1$, $\delta_{02} = \frac{1}{2}$ and $\delta_{03} = \frac{1}{2}$. Putting the value of $\delta_1, \delta_2, \delta_3, \delta_{01}, \delta_{02}$ and δ_{03} in (3.11), the corresponding optimal dual value i.e.,

$\qquad v(\delta) = (\frac{10}{1})^1(\frac{5}{1/2})^{1/2}(\frac{10}{1/2})^{-1/2}(\frac{6}{1})^1(\frac{10}{1/2})^{1/2}(\frac{5}{1/2})^{-1/2}$

$\qquad\qquad = 60$.

So for primal decision variables, the following equations found

$$10x_{11} = \delta_1\sqrt{v(\delta)} = 1 \times 7.75,$$

$$5x_{11}x_{12} = \delta_2\sqrt{v(\delta)} = \frac{1}{2} \times 7.75,$$

$$10x_{11}^3 x_{12} = \delta_3\sqrt{v(\delta)} = \frac{1}{2} \times 7.75,$$

$$6x_{21} = \delta_1\sqrt{v(\delta)} = 1 \times 7.75,$$

$$10x_{21}x_{22} = \delta_2\sqrt{v(\delta)} = \frac{1}{2} \times 7.75, \tag{3.13}$$

$$5x_{22}^3 x_{22} = \delta_3\sqrt{v(\delta)} = \frac{1}{2} \times 7.75.$$

From (3.13), optimal solutions are

$$x_{11} = 0.775, x_{12} = 1,$$

$$x_{21} = 1.292, x_{12} = 0.300.$$

Corresponding optimal value is

$$Z(x) = 15.366.$$

4. CONSTRAINED PROBLEM

4.1. Signomial geometric programming (SGP) problem:

4.1.1. Primal program:

A constrained primal signomial geometric programming (SGP) programming problem is of the form

Minimize $g_0(x_1, x_2, \ldots \ldots \ldots \ldots, x_m)$

Subject to $g_k(x_1, x_2, \ldots \ldots \ldots \ldots, x_m) \leq \zeta_k,$ k = 1, 2, ...,p

$\quad\quad\quad x_j > 0,$ j = 1, 2,......,m.

$\quad\quad$ Where $g_k(x) = \sum_{i=1}^{k} \sigma_i c_i \prod_{j=1}^{m} x_j^{a_{ij}},$ and $\zeta_k = \pm 1.$ $\tag{4.1}$

4.1.2. Dual program:

Dual geometric program (GP) problem of the given primal signomial program (SGP) problem is

Maximize $\zeta_0 \left[\prod_{i=1}^{n}(\frac{c_i}{\delta_i})^{\sigma_i\delta_i} \prod_{k=1}^{p} \lambda_k^{\zeta_k\lambda_k}\right]^{\zeta_0}$ $\tag{4.2}$

Subject to $\sum_{i=1}^{k} \sigma_i\delta_i = \zeta_k\lambda_k,$ $\quad\quad\quad\quad$ k = 0,1, ..., p

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$$\sum_{i=1}^{k} \sigma_i a_{ij} \delta_i = 0, \qquad\qquad j = 1,2, \dots\dots\dots, m$$

$$\delta_i > 0, \quad \lambda_0 = 1,$$

Case I: If $n \geq m+1$, (i.e. DD >0), then the dual signomial program presents a system of linear equations for the dual variables. A solution vector exists for the dual signomial variables.

Case II: If $n < m+1$, (i.e. DD <0), In this case generally no solution vector exists for the dual signomial variables. But using Least the Square (LS) or the Min-Max (MM) method one can get an approximate solution for this system.

Furthermore the primal-dual relation is

$$c_i \prod_{j=1}^{m} x_j^{*a_{ij}} = \zeta_0 \delta^*_i \, v(\delta^*, \lambda^*) \quad \text{for i} = 0,$$

And $\quad c_i \prod_{j=1}^{m} x_j^{*a_{ij}} = \delta^*_i / \lambda_k^* \quad$ for i $\epsilon [k]$, k$\geq$ 1 $\hfill$ (4.3)

Example (Constrained SGP problem) 4:

Min $g(x,y,z) = 2y^2 z^4 - 5x^2$

Subject to $\quad 2x^2 y^{-2} - y^{-1} z \leq -1$

$$x, y, z > 0.$$

Solution:

Assume $\zeta_0 = 1$,

Corresponding dual of the given primal is

Maximize v(δ)= $(\frac{2}{\delta_1})^{\delta_1} (\frac{5}{\delta_2})^{-\delta_2} (\frac{2}{\delta_{01}})^{\delta_{01}} (\frac{1}{\delta_{02}})^{-\delta_{02}} \lambda_1^{-\lambda_1}$ $\hfill$ (4.4)

Subject to $\quad \delta_1 - \delta_2 = 1,$

$$-2\delta_2 + 2\delta_{01} = 0,$$

$$2\delta_1 - 2\delta_{01} + \delta_{02} = 0,$$

$$4\delta_1 - \delta_{02} = 0, \hfill (4.5)$$

$$\lambda_1 = -\delta_{02} + \delta_{02}$$

$$\delta_1, \delta_2, \delta_{01}, \delta_{02} > 0.$$

From (4.5), $\delta_1 = -\frac{1}{2}, \delta_2 = -\frac{3}{2}, \delta_{01} = \frac{-3}{2}$ and $\delta_{02} = -2$, but $\delta_i > 0$.

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i.e., $\zeta_0 = -1$,

Corresponding dual of primal is

Maximize $v(\delta) = -\left(\left(\frac{2}{\delta_1}\right)^{\delta_1}\left(\frac{5}{\delta_2}\right)^{-\delta_2}\left(\frac{2}{\delta_{01}}\right)^{\delta_{01}}\left(\frac{1}{\delta_{02}}\right)^{-\delta_{02}}\lambda_1^{-\lambda_1}\right)^{-1}$ (4.6)

Subject to $\delta_1 - \delta_2 = -1$,

$$-2\delta_2 + 2\delta_{01} = 0,$$

$$2\delta_1 - 2\delta_{01} + \delta_{02} = 0,$$

$$4\delta_1 - \delta_{02} = 0,$$

$$\lambda_1 = -\delta_{01} + \delta_{02} \quad\quad\quad\quad (4.7)$$

$$\delta_1, \delta_2, \delta_{01}, \ \delta_{02} > 0.$$

From (4.7), $\delta_1 = \frac{1}{2}, \delta_2 = \frac{3}{2}, \delta_{01} = \frac{3}{2}, \delta_{02} = 2$ and $\lambda_1 = \frac{1}{2}$. Putting the values of δ_1, δ_2 and

δ_{01} in (4.6) the corresponding optimal dual value

i.e., $v^*(\delta) = -0.349$.

From the primal-dual relation

$$2y^2z^4 = \zeta_0\delta_1 v^*,$$

$$5x^2 = \zeta_0\delta_2 v^*,$$

$$2x^2y^{-2} = \frac{\delta_{01}}{\lambda_1},$$

$$y^{-1}z = \frac{\delta_{02}}{\lambda_1}.$$

Taking logarithm of each side yields equations which are linear in the logarithms of the primal variables

ln 2 + 2ln y + 4lnz $= $ ln 0.175,

ln 5 + 2log x $=$ ln 0.522,

ln 2 + 2log x – 2log y $=$ ln 3,   (4.8)

$-$ ln y + ln z $=$ ln 4.

From (4.8), optimal solutions are

$$x = 0.323, y = 0.364, z = 1.456.$$

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4.2. Modified Signomial geometric programming (MSGP) programming:

4.1.3. Primal program:

A primal modified signomial GP programming problem is of the form

Minimize $g_0(x_{lj})$

Subject to $g_{lk}(x) \leq \zeta_{lk}$,   k = 1, 2, ...,p

$x_{lj} > 0$, j = 1, 2,......,m.

Where $g_{lk}(x) = \sum_{l=1}^{n} \sum_{i=1}^{k} \sigma_{li} c_{li} \prod_{j=1}^{m} x_{lj}^{a_{lij}}$, $\zeta_{00} = 0$ and $\zeta_{lk} = \pm 1$.   (4.9)

4.1.4. Dual program:

Dual GP problem of the given primal MSGP problem is

Maximize $\zeta_0 \left[\prod_{l=1}^{n} \prod_{i=1}^{k} (\frac{c_{li}}{\delta_{li}})^{\sigma_{li}\delta_{li}} \prod_{k=1}^{p} \lambda_k^{\zeta_k \lambda_k} \right]^{\zeta_0}$ (4.10)

Subject to $\sum_{i=1}^{k} \sigma_{li}\delta_{li} = \zeta_{00}$, $l = 0,1, ..., n$,

$\sum_{i=1}^{k} \sigma_{li} a_{ij}\delta_i = 0$, $j = 1,2, ..., m$

$\sum_{i=1}^{k} \sigma_{li}\delta_{li} = \zeta_k \lambda_k$,   k = 0,1, ..., p

$\delta_{li} > 0$, $\lambda_0 = 1$,

Case I: If nk$\geq$ nm+n, (i.e. DD >0), then the dual modified signomial program presents a system of linear equations for the dual variables. A solution vector exists for the dual signomial variables.

Case II: If nk<nm+n, (i.e. DD <0), In this case generally no solution vector exists for the dual signomial variables. But using the Least Square (LS) or the Min-Max (MM) method one can get an approximate solution for this system.

Furthermore from the primal-dual relation

$c_{li} \prod_{j=1}^{m} x_{lj}^{a_{lij}} = \zeta_0 \delta^*_{li} \sqrt[n]{v(\delta^*, \lambda^*)}$., $(l = 1,2, ..., k; i = 1,2, ..., n)$ for i = 0,

and

$c_{li} \prod_{j=1}^{m} x_{lj}^{a_{lij}} = \delta^*_{li} / \lambda_{lk}^*$ for i $\epsilon[k]$, k$\geq$ 1   (4.11)

Example (Constrained MSGP problem) 5:

Min Z(x) = $10x_{11} - 10x_{11}^3 x_{12} + 6x_{21} - 5x_{21}^3 x_{22}$

Subject to $x_{11}x_{12} + x_{21}x_{22} \leq 1$

$x_{11}, x_{12}, x_{21}, x_{22} > 0.$

Sol.

Taking $\zeta_0 = 1$,

Here the primal problem is

Min $Z(x) = 10x_{11} - 10x_{11}{}^3 x_{12} + 6x_{21} - 5x_{21}{}^3 x_{22}$

Subject to $x_{11}x_{12} + x_{21}x_{22} \leq 1$

$x_{11}, x_{12}, x_{21}, x_{22} > 0.$

And corresponding dual problem is

$$d(\delta) = (\tfrac{10}{\delta_{01}})^{\delta_{01}}(\tfrac{10}{\delta_{02}})^{-\delta_{02}}(\tfrac{6}{\delta_{11}})^{\delta_{11}}(\tfrac{5}{\delta_{12}})^{-\delta_{12}}(\tfrac{1}{\delta_{21}})^{\delta_{21}}(\tfrac{1}{\delta_{22}})^{\delta_{22}}(\delta_{21} + \delta_{22})^{(\delta_{21}+\delta_{22})} \quad (4.12)$$

Subject to

$\delta_{01} - \delta_{02} = 1,$

$\delta_{11} - \delta_{12} = 1,$

$\delta_{01} - 3\delta_{02} + \delta_{21} = 0,$

$-\delta_{02} + \delta_{21} = 0,$

$\delta_{11} - 3\delta_{12} + \delta_{22} = 0,$ (4.13)

$-\delta_{12} + \delta_{22} = 0.$

Approximate solutions of this system of linear equations are

$\delta_{01} = 2, \delta_{02} = 1, \delta_{21} = 1,$

And $\delta_{11} = 2, \delta_{12} = 1, \delta_{22} = 1.$

Then the objective function is

$$d(\delta) = (\tfrac{10}{2})^2(\tfrac{10}{1})^{-1}(\tfrac{6}{2})^2(\tfrac{5}{1})^{-1}(\tfrac{1}{1})^1(\tfrac{1}{1})^1(2)^2$$

$$= 18. \quad (4.14)$$

And from primal-dual relation following system of equations gives optimal primal variables

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$$10x_{11} = 2.\sqrt{18} = 8.49,$$

$$10x_{11}{}^3 x_{12} = 1.\sqrt{18} = 4.25,$$

$$6x_{21} = 2.\sqrt{18} = 8.49,$$

$$5x_{21}{}^3 x_{22} = 1.\sqrt{18} = 4.25,$$

$$x_{11}x_{12} = \frac{1}{2},$$

$$x_{21}x_{22} = \frac{1}{2}.$$

Solving the system of equations, optimal solutions are

$$x_{11} = 0.849, x_{12} = 0.589,$$

$$x_{21} = 1.415, x_{22} = 0.353.$$

and the value of objective function is $Z(x) = 8.38$.

5. APPLICATION

5.1. Problem (Unconstrained SGP):

The demand (d) of an item is uniform at the rate of 10 units per month. The set-up cost (c_0) of a production run is Rs 20, and the inventory holding cost (c_h) is Rs 50 per item per month. If the shortages cost (c_s) is Rs 50 per item per month, determine economic lot size (q) for one run also determine what is inventory level (s) at the beginning of each month.

As stated, this problem can be formulated as the unconstrained SGP problem

$$\begin{cases} minimize & \dfrac{10.20}{q} + \dfrac{50s^2}{2q} + \dfrac{50(q-s)^2}{2q} \\ where & q > 0, s > 0. \end{cases}$$

It is unconstrained signomial geometric programming problem. The optimal solution is $q = 4, s = 2$ and minimum average total cost is Rs 100/month.

5.2. Problem (Unconstrained MSGP):

In a multi-item (n items) inventory model, demand of an item is uniform at the rate of d_i units per month. The set-up cost of a production run is Rsc_{0i}, and the inventory holding cost is Rsc_{hi} per item per month. If the shortages cost is Rsc_{si} per

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item per month, determine economic lot size (q_i) for one run also determine what is inventory level (s_i) at the beginning of each month.

As stated, this problem can be formulated as the unconstrained MSGP problem

$$\begin{cases} minimize & g(q,s) = \sum_{i=1}^{n} \dfrac{d_i c_{0i}}{q_i} + \dfrac{c_{hi}s^2}{2q_i} + \dfrac{c_{si}(q-s)^2}{2q_i} \\ where & q_i > 0, s_i > 0. \end{cases}$$

In particular here we assume $n = 2$ and input data of this problem is given below

i	d_i	c_{0i}	c_{hi}	c_{si}
1	10	20	50	50
2	15	10	125	25

And the out-put value is

i	q_i	s_i	$g(q,s)$
1	4.00	2.00	177.45
2	3.87	1.94	

5.3. Problem (Constrained SGP):

A constrained signomial geometric programming (SGP) problem is of the following form

$$\begin{cases} minimize & -2x_1 x_2 x_3{}^4 x_4{}^{-1} + x_2{}^{-1}x_3{}^{-1} + 5x_1{}^{0.5}x_4 \\ subject\ to & -x_4{}^{0.5} + x_2{}^{1/3}x_3 \leq -1 \\ where & x_1, x_2, x_3, x_4 > 0. \end{cases}$$

It is constrained signomial geometric programming problem. The optimal solution is $x_1 = 0.408$, $x_2 = 0.004$, $x_3 = 18.570$, $x_4 = 16$ and minimum value is 38.322.

5.4. Problem (Constrained SMGP):

A constrained modified signomial geometric programming (MSGP) problem is of the following form

$$\begin{cases} minimize\ g(x) = \sum_{i=1}^{n} a_i x_{1i} x_{2i} x_{3i}{}^4 x_{4i}{}^{-1} + b_i x_{2i}{}^{-1}x_{3i}{}^{-1} + c_i x_{1i}{}^{0.5}x_{4i} \\ subject\ to\ \sum_{i=1}^{n} -x_{4i}{}^{0.5} + x_{2i}{}^{1/3}x_{3i} \leq w \\ where\ x_1, x_2, x_3, x_4 > 0. \end{cases}$$

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It is constrained MSGP problem. In particular here we assume $n = 2$ and input data of this problem is given below

i	a_i	b_i	c_i	w
1	-2	1	5	-2
2	-1	1	3	

And the out-put value is

i	x_{1i}	x_{2i}	x_{3i}	x_{4i}	$g(x)$
1	0.408	0.004	18.57	16.00	69.09
2	0.716	0.006	7.663	16.00	

6. CONCLUSION:

In this paper we have developed signomial geometric programming (SGP) problem with positive or negative integral degree of difficulty. Modified form of signomial geometric programming technique has been demonstrated and some theorems have been derived here. In this research paper we have developed the technique only in a crisp environment but in future research of uncertainty in fuzzy non-linear programming (NLP) model, by using different type of fuzzy numbers (fuzzy coefficients, random fuzzy number, generalized fuzzy number) such as random fuzzy number, institutional fuzzy number or adaptive fuzzy demand rate analytically should be more challenging and interesting.

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