INDEPENDENT JOURNAL OF MANAGEMENT & PRODUCTION (IJM&P)
http://www.ijmp.jor.br v. 7, n. 1, January - March 2016
ISSN: 2236-269X
DOI: 10.14807/ijmp.v7i1.381

AN OTTO ENGINE DYNAMIC MODEL

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Submission: 07/10/2015
Accept: 23/10/2015

ABSTRACT

Otto engine dynamics are similar in almost all common internal combustion engines. We can speak so about dynamics of engines: Lenoir, Otto, and Diesel. The dynamic presented model is simple and original. The first thing necessary in the calculation of Otto engine dynamics, is to determine the inertial mass reduced at the piston. One uses then the Lagrange equation. Kinetic energy conservation shows angular speed variation (from the shaft) with inertial masses. One uses and elastic constant of the crank shaft, k. Calculations should be made for an engine with a single cylinder. Finally it makes a dynamic analysis of the mechanism with discussion and conclusions. The ratio between the crank length r and the length of the connecting-rod l is noted with landa. When landa increases the mechanism dynamics is deteriorating. For a proper operation is necessary the reduction of the ratio landa, especially if we want to increase the engine speed. We can reduce the acceleration values by reducing the dimensions r and l.

Keywords: Otto engine, Dynamics, Lagrange equation, Dynamic model, Shaft elastic constant.

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1. INTRODUCTION

The dynamic study of mechanisms Otto engine type is most important to predict how that will work in real engines.

Some Otto engine dynamic models were presented by: (AMORESANO, 2013; DAWSON, 2005; DE FALCO, 2013A-B; GUZZELLA, 2004; HEYWOOD, 1988; PETRESCU, 2005, 2009, 2012a-b, 2014a-c, 2015; RAMOS, 1989).

Almost a quarter of the planet's population works directly or indirectly for the construction of machines. Most specialists are involved in the development and production of road vehicles.

If Otto engine production would stop right now, they will still working until at least about 40-50 years to complete replacement of the existing fleet today.

Old gasoline engines carry us every day for nearly 150 years. “Old Otto engine” (and his brother, Diesel) is today: younger, more robust, more dynamic, more powerful, more economical, more independent, more reliable, quieter, cleaner, more compact, more sophisticated, more stylish, more secure, and more especially necessary and wanted. At the global level we can manage to remove annually about 60,000 cars. But annually appear other million cars (see the table 1).

Table 1. World cars produced

year	**cars produced**
2011	59,929,016
2010	58,264,852
2009	47,772,598
2008	52,726,117
2007	53,201,346
2006	49,918,578
2005	46,862,978
2004	44,554,268
2003	41,968,666
2002	41,358,394
2001	39,825,888
2000	41,215,653
1999	39,759,847

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In full energy crisis since 1970 until today, production and sale of cars equipped with internal combustion heat engines has skyrocketed, from some millions yearly to over sixty millions yearly now, and the world fleet started from tens of millions reached today the billion. As long as we produce electricity and heat by burning fossil fuels is pointless to try to replace all thermal engines with electric motors, as loss of energy and pollution will be even larger. However, it is well to continuously improve the thermal engines, to reduce thus fuel consumption. Planet supports now about one billion motor vehicles in circulation.

Otto and diesel engines are today the best solution for the transport of our day-to-day work, together and with electric motors.

Even in these conditions internal combustion engines will be maintained in land vehicles (at least), for power, reliability and especially their dynamics.

2. DETERMINING THE FIRST EQUATIONS

The first thing necessary in the calculation of Otto engine dynamics, is to determine the inertial mass reduced at the piston (1).

$$\begin{cases} M \equiv M^* = m_t + m_{bA} \cdot \dfrac{r^2}{s'^2} + \dfrac{J_1}{s'^2} + \dfrac{J_2}{s'^2} \cdot \dfrac{\lambda^2 \cdot \cos^2\varphi}{\cos^2\alpha} \\ M = m_t + [(m_{bA} + \dfrac{J_1}{r^2}) \cdot (1 - \lambda^2 \cdot \sin^2\varphi) + \dfrac{J_2}{l^2} \cdot \cos^2\varphi] \cdot \dfrac{1}{\sin^2\varphi \cdot (\cos\alpha + \lambda \cdot \cos\varphi)^2} \\ M = m_t + \dfrac{m_1 \cdot (1 - \lambda^2 \cdot \sin^2\varphi) + m_2 \cdot \cos^2\varphi}{\sin^2\varphi \cdot (\cos\alpha + \lambda \cdot \cos\varphi)^2} \end{cases} \quad (1)$$

Then it derives the reduced mass to the crank position angle (2).

$$\frac{dM}{d\varphi} = (M - m_t) \cdot (-2) \cdot (\frac{\cos\varphi}{\sin\varphi} - \frac{\lambda \cdot \sin\varphi}{\cos\alpha}) - \frac{2 \cdot \cos\varphi \cdot (\lambda^2 \cdot m_1 + m_2)}{\sin\varphi \cdot (\cos\alpha + \lambda \cdot \cos\varphi)^2} \quad (2)$$

Lagrange equation is written in the form (3).

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$$M \cdot \omega^2 \cdot x'' + \frac{1}{2} \cdot \frac{dM}{d\varphi} \cdot \omega^2 \cdot x' = k \cdot (s - x) - F_p \tag{3}$$

Were used for piston the next kinematics parameters (4).

$$\begin{cases} s = r \cdot \cos\varphi + l \cdot \cos\alpha - l \\ s' = -\dfrac{r \cdot \sin\varphi}{\cos\alpha} \cdot (\cos\alpha + \lambda \cdot \cos\varphi) \\ s'' = -r \cdot \cos\varphi - \dfrac{r \cdot \lambda \cdot \cos(2\varphi)}{\cos\alpha} - \dfrac{r \cdot \lambda^3 \cdot \sin^2\varphi \cdot \cos^2\varphi}{\cos^3\alpha} \end{cases} \tag{4}$$

3. DYNAMIC EQUATIONS

The dynamic equation of motion of the piston, obtained by integrating the Lagrange equation (3), takes the form 5.

$$x = s \cdot \sqrt[3]{\frac{k}{k - m_t \cdot \omega^2}} - c_3 \cdot \frac{\cos\varphi}{\cos\alpha \cdot (\cos\alpha + \lambda \cdot \cos\varphi)} + c_4 \cdot \cos\varphi \tag{5}$$

Dynamic reduced velocity (6) and dynamic reduced acceleration (7) are obtained by derivation:

$$x' = s' \cdot \sqrt[3]{\frac{k}{k - m_t \cdot \omega^2}} + c_3 \cdot \frac{\sin\varphi}{\cos\alpha \cdot (\cos\alpha + \lambda \cdot \cos\varphi)} - c_4 \cdot \sin\varphi \tag{6}$$

$$x'' = s'' \cdot \sqrt[3]{\frac{k}{k - m_t \cdot \omega^2}} + c_3 \cdot \frac{\cos\varphi}{\cos\alpha \cdot (\cos\alpha + \lambda \cdot \cos\varphi)} - c_4 \cdot \cos\varphi \tag{7}$$

Angular velocity ω^* is obtained through kinetic energy conservation (8-12).

$$\frac{1}{2} \cdot J^* \cdot \omega^{*2} = \frac{1}{2} \cdot J_D^* \cdot \omega_D^2 \tag{8}$$

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$$\begin{cases}\omega_D = \omega_m \cdot D = \omega_m \cdot (\cos\alpha)^2 = \omega_m \cdot \cos^2\alpha = \\ \omega_m \cdot (1-\sin^2\alpha) = \omega_m \cdot (1-\lambda^2 \cdot \sin^2\varphi)\end{cases} \quad (9)$$

$$J^* = J_1 + m_{bA} \cdot r^2 + m_t \cdot s'^2 \quad (10)$$

$$J_D^* = J_1 + m_{bA} \cdot r^2 + m_t \cdot x'^2 \quad (11)$$

$$\omega^* = \sqrt{\frac{J_1 + m_{bA} \cdot r^2 + m_t \cdot x'^2}{J_1 + m_{bA} \cdot r^2 + m_t \cdot s'^2}} \cdot \frac{\pi \cdot n}{30} \cdot (1-\lambda^2 \cdot \sin^2\varphi) \quad (12)$$

Dynamic velocity (13) and kinematics velocity (14) are written:

$$\dot{x} = x' \cdot \omega^* \quad (13)$$

$$\dot{s} = s' \cdot \omega_m = s' \cdot \frac{\pi \cdot n}{30} \quad (14)$$

Dynamic acceleration (15) and kinematics acceleration (16) are written:

$$\ddot{x} = x'' \cdot \omega^{*2} \quad (15)$$

$$\ddot{s} = s'' \cdot \omega_m^2 = s'' \cdot \frac{\pi^2 \cdot n^2}{900} \quad (16)$$

4. NOTATIONS AND FIGURES

In the Figure 1 it presents the crank shaft.

The relation (17) determines the elastic constant of the crank shaft, k.

$$k = \frac{3 \cdot \pi \cdot E \cdot G \cdot (D_m^4 - d_m^4)}{4G(l_m + b)^3 + 96Er^2 \sin^2\varphi (D_m^4 - d_m^4)[\frac{l_p + .4D_p}{D_p^4 - d_p^4} + \frac{l_m + .4D_m}{D_m^4 - d_m^4} + \frac{8r - 1.6(D_p + D_m)}{b(2r + D_p + D_m)^3}]} \quad (17)$$

For the masses one uses the notations (18); see the Figure 2.

$\lambda \Rightarrow$ the ratio between lengths of crank and rod; $\lambda = \frac{r}{l}$

ISSN: 2236-269X
DOI: 10.14807/ijmp.v7i1.381

m_p $\Rightarrow$ the mass of the piston, with piston bolt and segments;

m_b $\Rightarrow$ the mass of the rod;

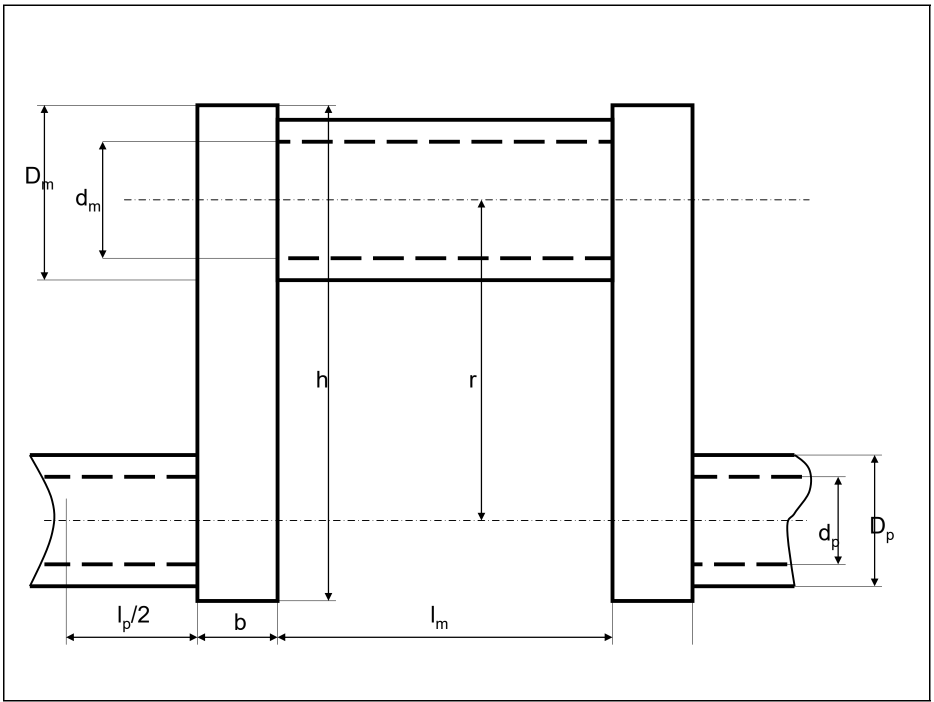

Figure 1: Crank Shaft

$$\begin{cases} m_{bA} = m_b \cdot \dfrac{l''}{l} \quad m_{bB} = m_b \cdot \dfrac{l'}{l} \quad l' + l'' = l \quad m_{bA} + m_{bB} = m_b \\ m_t = m_p + m_{bB} \\ m_1 = m_{bA} + \dfrac{J_1}{r^2} \\ m_2 = \dfrac{J_2}{l^2} \end{cases} \tag{18}$$

The parameters c1-c4 take the forms (19):

$$\begin{cases} c_1 = \dfrac{r}{k} \cdot \omega^2 \quad [\dfrac{m}{kg}] \\ c_2 = \lambda^2 \cdot m_1 + m_2 \quad [kg] \\ c_3 = c_1 \cdot c_2 \quad [m] \\ c_4 = c_1 \cdot m_t \quad [m] \end{cases} \tag{19}$$

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The moment of inertia J_1 can be determined with the relation (20).

$$J_1 = \frac{\pi \cdot \rho}{32} \cdot \{(l_p + 2 \cdot b) \cdot (D_p^4 - d_p^4) + (l_m + 2 \cdot b) \cdot [(D_m^4 - d_m^4) + (D_m^2 - d_m^2) \cdot 8 \cdot r^2]\} \tag{20}$$

The crank length, r, and the length of the connecting-rod, l, can be seen in the kinematics schema of an Otto mechanism (Figure 2).

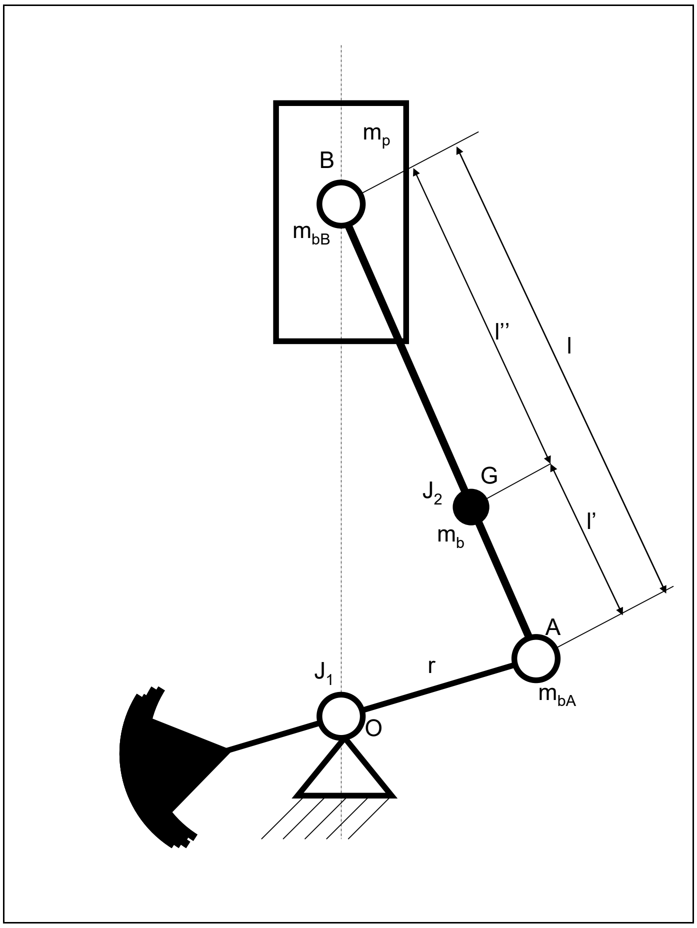

Figure 2: Otto mechanism kinematics schema

5. DYNAMIC ANALYSIS OF THE MECHANISM, DISCUSSION AND CONCLUSION

When λ increases the mechanism dynamics is deteriorating.

r=0.25 [m] l=0.3 [m] $\lambda = 0.8(3)$

For n=8000 [r/m] the mechanism is working normally (see the accelerations diagram from the picture 3):

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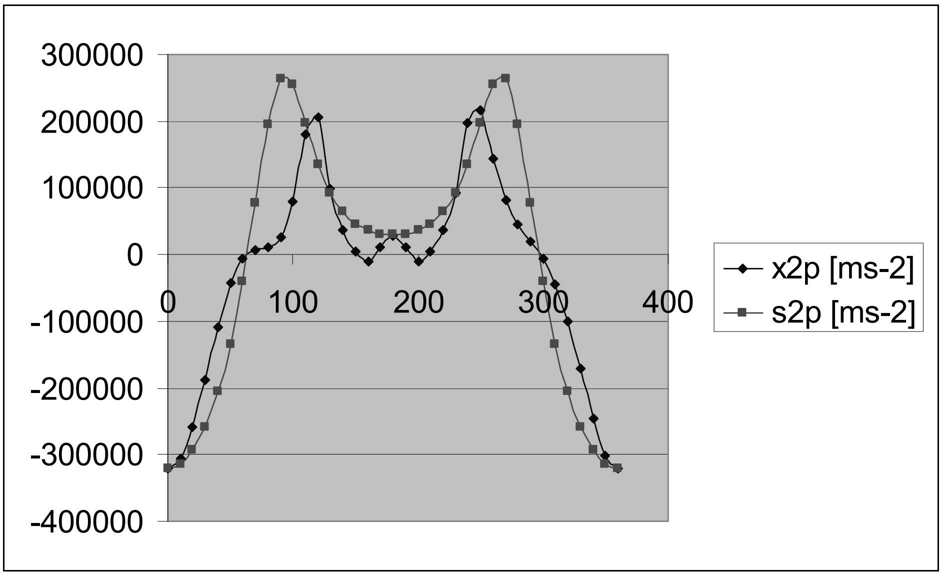

Figure 3: Dynamic and kinematics accelerations; n=8000 [r/m]; $\lambda = 0.83$

r=0.25 [m] l=0.3 [m] $\lambda = 0.8(3)$

At n=9000 [r/m] the mechanism work abnormally (see the accelerations diagram from the picture 4):

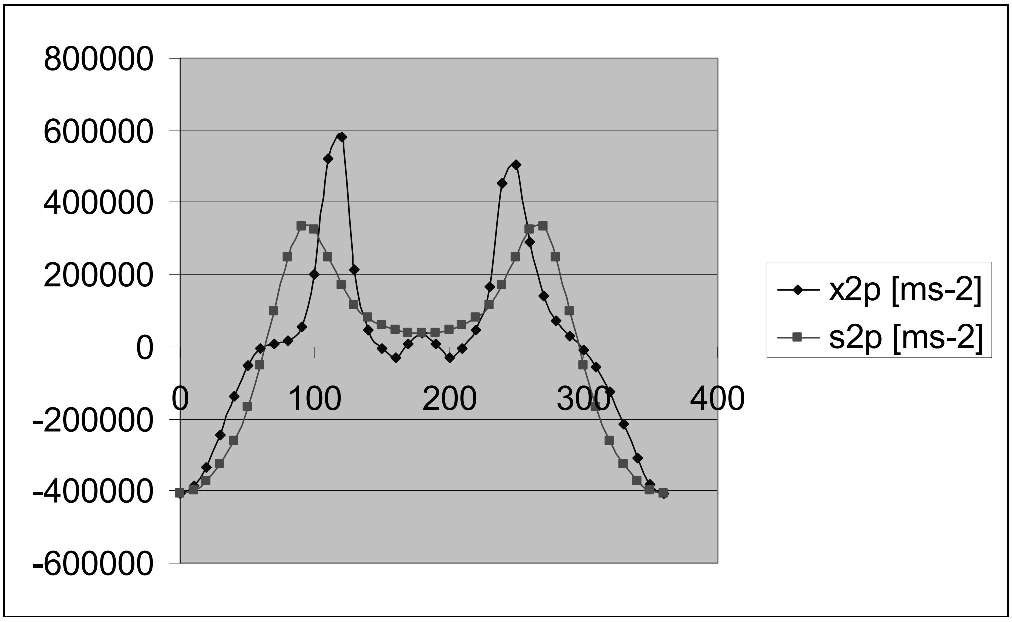

Figure 4: Dynamic and kinematics accelerations; n=9000 [r/m]; $\lambda = 0.83$

r=0.25[m];l=0.3[m]

For a proper operation is necessary reduction of the ratio λ, especially if we want to increase the engine speed (see the next diagrams).

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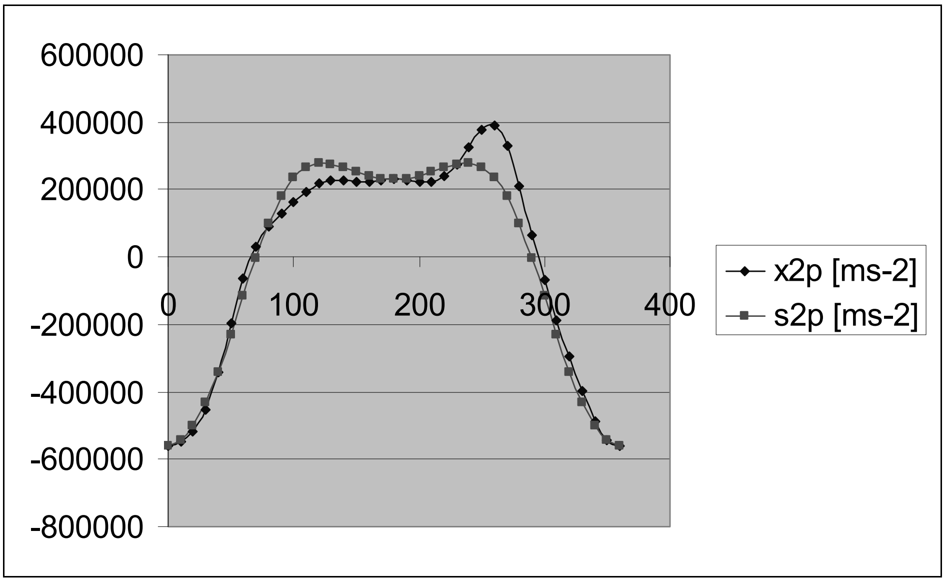

Figure 5: Dynamic and kinematics accelerations; n=12000 [r/m];

r=0.25[m];l=0.6[m] $\lambda = 0.42$

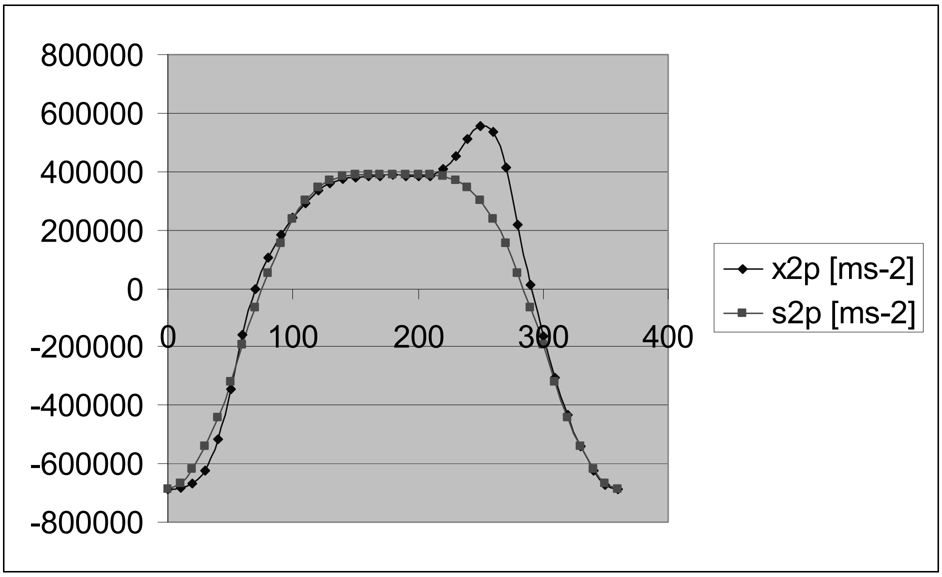

Figure 6: Dynamic and kinematics accelerations; n=14000 [r/m];

r=0.25[m];l=0.9[m] $\lambda = 0.27$

We can reduce the acceleration values by reducing r and l.

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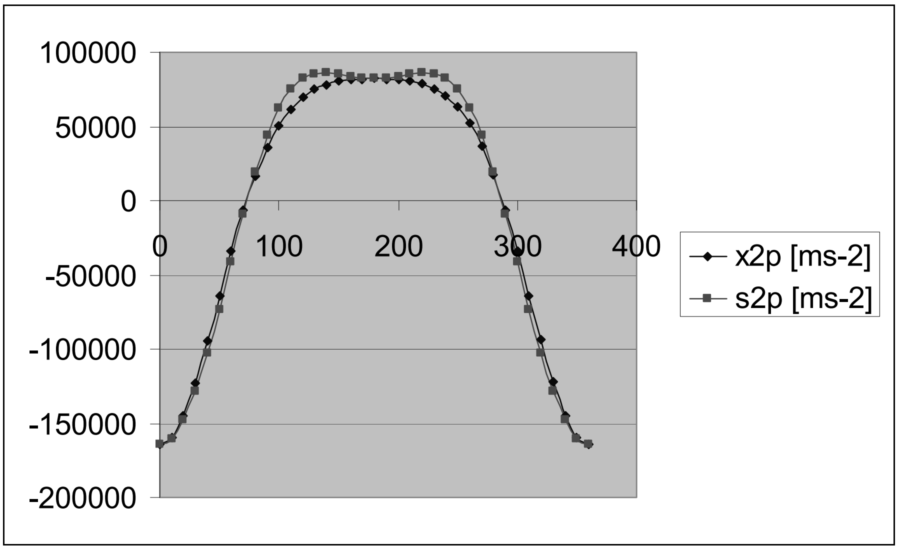

Figure 7: Dynamic and kinematics accelerations; n=15000 [r/m];

r=0.05[m];l=0.15[m] $\lambda = 0.33$

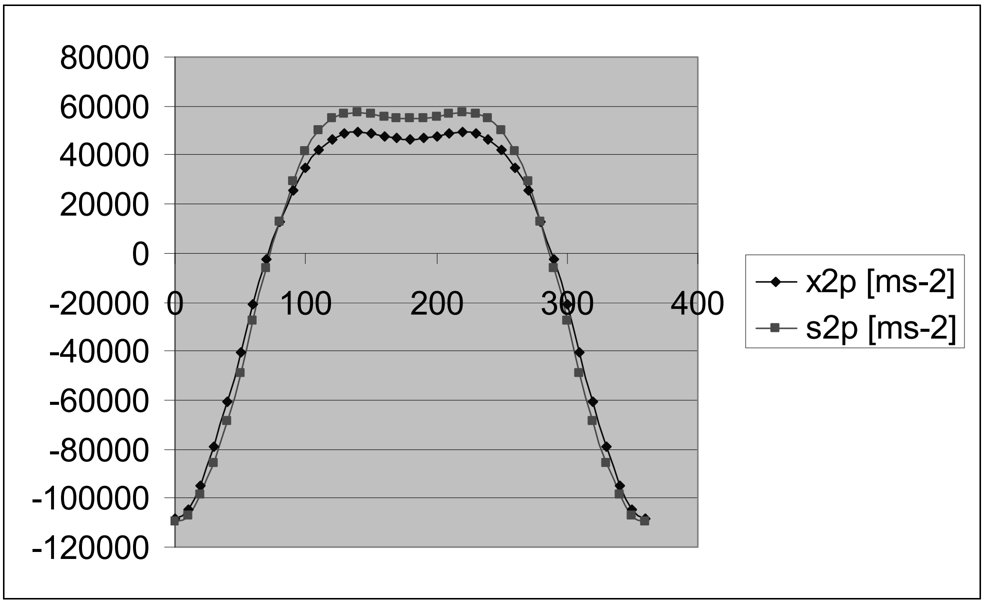

Figure 8: Dynamic and kinematics accelerations; n=50000 [r/m];

r=0.003[m];l=0.009[m] $\lambda = 0.33$

One can reduce the acceleration values especially if we want to increase the engine speed by reducing r and l (the lengths of crank and rod).

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